ANALYSIS OF TEMPERATURE STRESSES IN ACTIVE ELEMENTS OF A LASER

B. R. Belostotskii

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A method of analyzing the temperature stresses in active elements of optical quantum generators operating in the pulsed and continuous modes is elucidated.

The presence of a nonuniform temperature field in the active body of an optical quantum generator (laser) specifies the appearance of temperature stresses which, in turn, result often in significant deformation and fissuring of the sample, and degradation of its characteristics.

Let us examine the active element of a laser, of circular cylindrical shape. The existence of three principal normal stresses, radial σ_r, tangential σ_θ, and axial σ_z, results from an analysis of the equilibrium conditions of a volume element of the circular cylinder cut in the shape of an annular sector. If it is assumed that the active substance is an isotropic medium, the thermophysical and strength characteristics of which (β the coefficient of linear temperature expansion; ν the Poisson ratio; and E the elastic modulus) are independent of the temperature, then the magnitudes of the mentioned stresses are determined in general form from the relationships [1]:

$$\sigma_r = \frac{\beta E}{2(1-\nu)} \{[\bar{\vartheta}(\mathrm{Fo})]_{0\to 1} + [\bar{\vartheta}(\mathrm{Fo})]_{0\to r_1}\}, \tag{1}$$

$$\sigma_0 = \frac{\beta E}{2(1-\nu)} \{-2\vartheta(r_1, \mathrm{Fo}) + [\bar{\vartheta}(\mathrm{Fo})]_{0\to 1} + [\bar{\vartheta}(\mathrm{Fo})]_{0\to r_1}\}, \tag{2}$$

$$\sigma_z = \frac{\beta E}{1-\nu} \{-\vartheta(r_1, \mathrm{Fo}) + \nu [\bar{\vartheta}(\mathrm{Fo})]_{0\to 1}\} + \varepsilon_z E, \tag{3}$$

where $[\bar{\vartheta}(\mathrm{Fo})]_{0\to r_1}$ is the mean-volume excess temperature of a cylinder of radius r_1 written as follows:

$$[\bar{\vartheta}(\mathrm{Fo})]_{0\to r_1} = \frac{2}{r_1^2} \int_0^{r_1} \vartheta(r_1, \mathrm{Fo})\, r_1 dr_1. \tag{4}$$

In the approximation of plane deformation, i.e., upon compliance with the condition

$$\int_0^1 r_1 \sigma_z dr_1 = 0, \tag{5}$$

it is easy to show that the following equalities are valid at any time:

at any point of the cylinder

$$\sigma_z = \sigma_z + \sigma_\theta, \tag{6}$$

on the cylinder surface (for $r_1 = 1$)

$$\sigma_r = 0, \quad \sigma_z = \sigma_\theta, \tag{7}$$

at the center of the cylinder (for $r_1 = 0$)

$$\sigma_r = \sigma_\theta. \tag{8}$$

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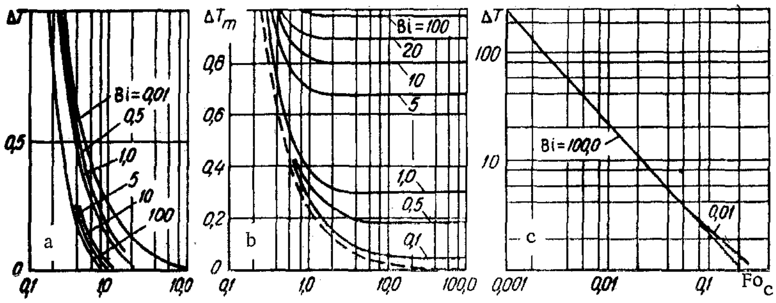

Fig. 1. Temperature drop in fractions of ϑ_{imp} between the center and surface of the active element of a laser in the quasistationary mode: a, c) temperature drop ΔT for $Fo = Fo_c$; b, c) maximum temperature drop ΔT_m (dashed curve corresponds to the temperature drop for a CW laser).

For practical purposes it is interesting to estimate the maximum stresses. They can originate during the formation of a maximum temperature difference between the center and the surface. Determination of this latter is fraught with great difficulties since it is not known for which value of Fo the temperature stress calculation should be executed.

As has been established earlier [2], the quasistationary temperature mode of the active element of a pulsed laser is characterized by the fact that for quasicontinuous pulses the same temperature field is reproduced over the sample cross section to the end of each cycle. During adiabatic homogeneous pumping, the temperature of each point of the sample increases by the same quantity ϑ_{imp}, i.e., no change is observed in the profile of the temperature field under the mentioned conditions of pumping progress. During cooling, the temperature field varies in both magnitude and profile:

$$\vartheta(r_1, \mathrm{Fo}) = \sum_{n=1}^{\infty} \frac{2\vartheta_{imp} \mathrm{Bi} J_0(\mu_n r_1) \exp(-\mu_n^2 \mathrm{Fo})}{J_0(\mu_n)[\mu_n^2 + \mathrm{Bi}^2][1 - \exp(-\mu_n^2 \mathrm{Fo_c})]}. \tag{9}$$

The deduction can be made that the profile of the temperature field at the beginning and ending of the cooling period will be the same if, within one cycle, the temperature difference between the center and the surface of the active body first starts to increase, reaches a maximum value, and then again decreases.

Attention is not ordinarily turned to this fact. Computations are carried out for $Fo = Fo_c$; the results often turn out to be lowered, and the strength condition for the sample may not even be satisfied in practice.

By using the Minsk-22 electronic computer in conjunction with Eq. (9), we succeeded in tracing the change in the temperature drop between the center and the surface of the active body within a cycle in the quasistationary mode for a number of Biot values. The computation was performed for $n = 6$; the numerical values of the eigennumbers μ_n were taken from a table presented in the monograph [3]. The results of the computations are presented in Fig. 1.

As is seen from Fig. 1, the discrepancy in the quantities ΔT_m and ΔT can depend essentially on the effectiveness of the cooling system used and the cycle duration. The greatest discrepancies are realized for $\mathrm{Bi} \to \infty$. As the cycle duration diminishes, the dependence of the difference $[\Delta T_m - \Delta T]$ on the number Fo_c is much weaker, and for $Fo_c < 0.01$ is practically absent. For $Fo_c < 0.01$, the quantity ΔT_m is practically independent of the number Bi also. This means that the temperature mode for quasicontinuous pulsing is equivalent to the temperature mode of a CW laser: in the continuous mode the quantity ΔT is independent of the cooling system efficiency and is defined by [4]

$$\Delta T = \frac{1}{4\mathrm{Fo_c}}. \tag{10}$$

Therefore, in this case the temperature stresses are determined by means of the expressions

$$\sigma_r = -\frac{\beta E \vartheta_{imp} \Delta T}{4(1-\nu)}(1 - r_1^2), \tag{11}$$

$$\sigma_\theta = -\frac{\beta E \vartheta_{\mathrm{imp}} \Delta T}{4(1-\nu)}(1-3r_1^2), \tag{12}$$

$$\sigma_z = -\frac{\beta E \vartheta_{\mathrm{imp}} \Delta T}{2(1-\nu)}(1-2r_1^2). \tag{13}$$

For Fo > 0.1 it is impossible to use (11)-(13) since the results obtained are lowered (see the location of the dashed curve in Fig. 1b). Using (1)-(3) with (9) taken into account is difficult, especially for cases of taking account of the nonuniformity in pumping and heat exchange during pumping, as well as because of the need to determine the time Fo at which ΔT_m is realized. The problem is simplified somewhat for calculations by means of relationships from approximate solutions obtained by using a variational method, say [5, 6]:

$$\vartheta(r_1, \mathrm{Fo}) = PB\vartheta_{\mathrm{imp}} \frac{\exp(-k\,\mathrm{Fo})}{1-\exp(-k\,\mathrm{Fo_c})} \tag{14}$$

where

$$B = \frac{\mathrm{Bi}}{2+\mathrm{Bi}}; \quad k = \frac{8\mathrm{Bi}}{4+\mathrm{Bi}}; \quad P = \frac{1-0.5B}{1-B+\dfrac{B^2}{3}}. \tag{15}$$

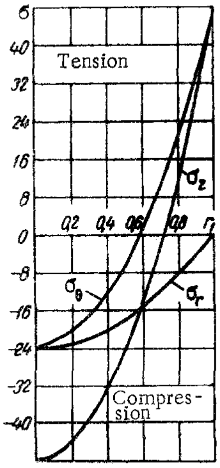

Fig. 2. Temperature stress distribution over the radius of the active body (σ, kg/cm²).

The method of determining the integration constants in this case reduces to the fact that for the beginning of the cooling period, i.e., for Fo = 0, the temperature difference between the center and the surface turns out to be somewhat greater as compared with the corresponding quantities obtained from the exact solutions. The quantity ΔT is determined from the expression

$$\Delta T = \frac{PB}{1-\exp(-k\mathrm{Fo_c})}. \tag{16}$$

Taking account of the quite approximate values, as a rule, of the magnitudes of the thermophysical and strength characteristics of active materials, the last remark turns out to be quite essential and affords the possibility of recommending the use of Eqs. (11)-(13), taking account of (16), for the determination of the temperature stresses.

In Fig. 2 we present as an illustration the results of computing the temperature stresses in a specific sample of neodymium glass (length 75 mm, diameter 4 mm) with water cooling (Bi = 15) at a pulse repetition rate f = 0.1 Hz ($\mathrm{Fo_c} \approx 0.1$). It was assumed in the computations that $E = 6.6 \cdot 10^5$ kg/cm²; $\nu = 0.25$; $\beta = 1.08 \cdot 10^{-5}$ deg^{-1}. Calorimetry of the active body determined the temperature jump during the pumping as $\vartheta_{\mathrm{imp}} = 4$°C.

NOTATION

$r_1 = r/R$	is the dimensionless running radius of the cylinder;
ε_z	is the relative strain along the cylinder axis;
$\mathrm{Fo} = a\tau/R^2$, $\mathrm{Fo_c} = a\tau_c/R^2$	are the Fourier numbers for the times τ and τ_c;
τ	is the running time;
τ_c	is the time between alternate pumping pulses (cycle duration);
$\mathrm{Bi} = \alpha R/\lambda$	is the Biot number.

The remaining notation is taken from [3].

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